Article

The Newton-Leibniz Priority Dispute

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Summary: *In the early eighteenth century a priority dispute erupted. The President of the Royal Society, Sir Isaac Newton, and one of the foremost continental philosophers at the time, Gottfried Wilhelm Leibniz, both claimed priority in the discovery of the differential and integral calculus. We now credit Newton as the first discoverer of the calculus, Leibniz being a close second. This review, based on a public lecture delivered by the author in October 2000, will examine the historical and mathematical aspects of the dispute.*

England in the early eighteenth century saw the beginning of perhaps the most famous dispute in the history of Mathematics. In truth, its seeds were sown in the previous century. One of the contenders, Sir Isaac Newton, then President for Life of the Royal Society, had an important matter in hand and delayed the dispute by nearly three decades. But before we go into the dispute let us familiarise ourselves with the two main characters.

Two lives

Sir Isaac Newton was born prematurely in a manor house in Woolsthorpe, Lincolnshire on Christmas Day 1642 (Julian calendar), the same year Galileo died. His father died before he was born. His mother remarried the Reverend Barnabas Smith, an elderly rector. Newton was born into a family of some means, which were certainly increased by this marriage. However, Newton was sent to live with his maternal grandparents. He returned to his maternal home when his stepfather died eight years later. At the age of 12, Isaac attended the grammar school at Grantham, where he lodged with an apothecary, Mr. Clarke. That may have been his source of interest in alchemy.

He joined Trinity College, Cambridge, in 1661 at the age of 19. Three years later he was elected a scholar of Trinity, a year after which he became a Bachelor of Arts. In 1665 the plague reached Cambridge and Newton temporarily returned to his native Lincolnshire to avoid it. The three years he spent in his family home in Lincolnshire are known among Newtonian scholars as the *anni mirabiles*. There he discovered the Binomial Theorem to any power and in November 1665 the Differential Calculus, or as he referred to it, the Method of Fluxions. The Integral Calculus, or Quadrature of Curves, he discovered in May 1666, after discovering the Theory of Colours. The Law of Gravity and the orbits of planets followed.

After his return to Trinity College in the spring of 1667 he was elected a Fellow. Two years later, at the age of 26, he became the Lucasian Professor of Mathematics. Newton secretly pursued the study of alchemy and theology, apart from mathematics and dynamics.

Meanwhile his discoveries were as yet unpublished. He sent to John Collins, a mathematical impresario, the *De Analysi*, a document in which he set out his results on infinite series and the calculus. We owe the belated printing of Newton's *magnum opus* to the astronomer Dr. Edmund Halley. He visited Newton in 1684 and asked him what course a planet would follow if the force of attraction between sun and planet followed an inverse square law. Newton answered "an ellipse", and sent him the proof the following November in a paper called *De Motu*. This paper was expanded into the *Principia* which was published on the 5th of July1686. Significantly, he did not derive any of his results by means of his calculus.

In 1696 Newton became Warden of the Mint. He undertook his new position with zeal and was an able administrator. His duties included the persecution of forgers. In 1700 he was appointed Master of the Mint. In 1701 he gave up the Lucasian Chair. In November 1703 he was elected President of the Royal Society. And here we leave Newton for a while and proceed with a brief life of Leibniz [1].

Gottfried Wilhelm Leibniz was born in Leipzig in 1646. His father Friedrich, was a professor of moral philosophy at Leipzig. He died when Gottfried was six. His mother Catharina Schmuck, was Friedrich's third wife. He entered school when he was seven. There he learned Latin and Greek, the knowledge of which he supplemented by further reading. At the age of fourteen he entered the University of Leipzig where he studied philosophy and mathematics. He was also taught Latin, Greek and Hebrew. He was awarded a bachelor's degree in 1663.

Leibniz then proceeded to Jena. There the professor of mathematics, Erhard Weigel, who was also a philosopher, influenced Leibniz. Back in Leipzig he obtained a master's degree in philosophy. A few days later his mother died. After being refused a doctorate in law, ostensibly because of his youth, he went to the University of Altdorf. He received his doctorate in 1667.

He then went to Frankfurt, where he lived for a few years under the employ of Baron Johann Christian von Boineburg. His task included that of a secretary, librarian, and a lawyer. There he pursued various projects of a scientific, literary and political nature. He also served as lawyer to the courts of Mainz.

In 1671 he published a book on Physics, *Hypothesis Physica Nova.* In 1672 Boineburg sent Leibniz on a diplomatic mission in Paris. While in Paris he availed himself of the opportunity to make contact with French mathematicians and other intellectuals. Christian Huygens, astronomer and mathematician, was perhaps the most important contact.

The following year, Leibniz accompanied Boineburg's son and nephew on a mission to London. While in London, Leibniz met many English intellectuals, including Robert Hooke, Robert Boyle and John Pell. He was also elected a member of the Royal Society after giving a demonstration of his as yet unfinished calculating machine. It was at this time that Leibniz became acquainted with John Collins and Henry Oldenburg

Back in Paris, Leibniz studied extensively mathematics under Huygens – his visit to London convinced him that his knowledge of the subject was not adequate. After various efforts in formulating the differential and integral calculus in November 1675 he wrote a manuscript where he used the $\int f(x)dx$ notation for the first time. In autumn 1676 he discovered the now familiar result $d(x^n) = nx^{n-1}dx$ for integral values of n.

In 1676 Leibniz revisited London. He visited Collins and, without the author's knowledge, was shown Newton's paper *De analysi*. Although he took thirteen pages of notes on series he did not jot one note on the calculus. It is possible that he may have already discovered his method of calculus. The President of the Royal Society, Henry Oldenburg and Collins, who marvelled at Leibniz' mathematical abilities, persuaded Newton to correspond with Leibniz in the same year. He sent two letters, the *Epistula prior* and the *Epistula posterior*. In the former Newton expounded his theory of infinite series and the binomial theorem. In the second letter Newton expounded further the theory of infinite series. He "discussed" the calculus in a discrete way and in anagrams. He revealed nothing but simply indicated that he had a method of finding tangents and areas. Then Oldenburg died and the correspondence ceased.

In 1676 Leibniz was offered the post of librarian by the Duke of Hanover. He reluctantly left Paris in October to Hanover via London and Holland, where he met Spinoza. Hanover remained his home until his death, though he travelled extensively.

Another important achievement by Leibniz is the invention of the binary system and arithmetic in 1679. He also discovered determinants while trying to solve systems of linear equations. The latter discovery remained unpublished. Throughout the 1680's he composed and published several philosophical works.

In 1684 he published *Nova Methodus pro Maximis et Minimis, itemque Tangentibus...* in *Acta eruditorum*, a German journal he helped to found. In this paper he described the Differential Calculus, but gave no derivations for his results. In this paper Leibniz uses differentials *dx*, etc. rather than derivatives. Two years later in the same journal he published *De Geometria Recondita et Analysi Indivisibilium atque Infinitorum* where he expounded his Integral Calculus. There was no reference to Newton in either papers. But Newton was at the time busy with the publication of his *Principia* and the dispute was postponed.

The seeds of contention are sown

It is relevant to a history of the Newton-Leibniz dispute to examine how Newton came to allow his *De analysi* out of the confines of Cambridge. In 1669 Isaac Barrow, who at the time was the Lucasian professor of mathematics, received a book from John Collins. He showed Newton the book, which was entitled *Logarithmotechnica* by Nicholas Mercator. In the book was the series for $\log_e (1 + x)$, a series which Newton had already found. Barrow, without Newton's knowledge, sent Collins the *De Analysi.* The paper dealt with infinite series, quadratures and his method of fluxions. Collins sent back the paper only after he had the manuscript copied. Collins, without the author's permission, sent the manuscript to various mathematicians both in England and abroad. This is the very document which Leibniz saw on his second visit to Collins in 1676.

Newton and Collins met for the first time in November 1669. The two men kept up a correspondence until 1672. Newton then became more interested in alchemy. Although Newton would not have approved of Collins spreading his work throughout Europe, this worked in his favour. Collins died in 1683 after suffering for seven years from a terrible illness. In 1708 his papers were passed to William Jones. The *De analysi* was among them. While the priority dispute raged these papers were used by Newton to support his claim.

As mentioned above, Leibniz published two papers on the differential and integral calculus in 1684 and 1686, respectively. His first paper was referred to by his allies Johann and Jacob Bernoulli as 'an enigma rather than an explication'. He did not refer to Newton's work at all. He mentioned neither the *Epistula prior* nor the *Epistula* *posterior*. The *De Analysi*, which he perused at leisure when he visited Collins in London in 1676, he conveniently forgot to mention. But Halley's visit to Newton and the preparation for publication of the first edition of the *Principia* delayed the dispute.

Nicolas Fatio de Duillier (1664 – 1753) was a Swiss mathematician and a protégé of Newton. Fatio, whom Newton had known since 1689, was probably the closest friend Newton ever had. Their relationship cooled somewhat by 1693, the year when Newton had a mental breakdown. Yet in his treatise *A Double Geometrical Investigation into the Line of Quickest Descent* published in 1699, after asserting his independent discovery of the calculus, he wrote that Newton was the first to discover the Calculus and accused Leibniz of taking advantage of Newton's modesty

Leibniz was not provoked. He wrote that learned men should not fight like fishwives, and that Newton would not approve of such rubbish that Fatio wrote. He added that Newton and himself were original masters of the calculus, as their success in solving problems on maxima and minima has shown, and which Newton had already demonstrated in 1687, that is, after Leibniz' 1684 paper!

Earlier in 1693 Leibniz sent a letter to Newton which was very courteous. Newton answered in similar terms, and wrote that he valued very highly his friendship with "one of the leading geometers of this century" and begged his censure on any point, since "I value friends more than mathematical discoveries." He also sent him a general solution using fluxional notation. This does not mean that Newton was not anxious about losing priority in the discovery of the calculus. In the first edition of the *Principia* (July 1686), Newton, perhaps troubled by the knowledge that Leibniz had also discovered the calculus, asserted that he corresponded with "that most excellent geometer, G.W. Leibniz" and that Leibniz communicated his method which hardly differed from Newton's "except in his forms of words and symbols". By the third edition, as the dispute progressed, all reference to Leibniz in the *Principia* disappeared.

In 1699 Leibniz criticised, anonymously, David Gregory's demonstration of the catenary. He found an error and claimed that the fault in the demonstration lay in the shortcomings of Newton's fluxional method. Leibniz clutched to every straw at hand to defend his claim to priority. His habit of launching his attacks anonymously, however, only earned him derision.

In 1704 Newton finally published *De Quadratura* as an appendix to the *Opticks*, although in 1693 John Wallis published a brief account of fluxions in his book *Geometry*. In the Advertisement he mentions a letter which he wrote to Leibniz in which he describes "a Method by which I had found some general Theorems about squaring Curvilinear Figures". The following year, in the journal *Acta eruditorum*, Leibniz reviewed under cover of anonymity *Opticks* in which he compared Newton to Honoré Fabri, a man known for plagiarism. Fabri composed his geometry *Synopsis geometrica* (1669) by using a work by Francesco Cavalieri and by substituting different terminology claimed to have developed a new method. Similarly, according to Leibniz, Newton had used fluxions rather than differentials. Despite Leibniz' denial that he had no intention of accusing Newton of plagiarism his intention was obvious.

In 1710 John Keill in a paper on centrifugal forces in the *Philosophical Transactions* asserted Newton's priority and charged Leibniz with plagiarism. This accusation ushered in the second phase of the controversy. However, before we delve straight into an account of the controversy, let us examine our protagonists' respective methods.

Fluxions and differentials compared [4]

Newton's approach to the calculus was a dynamical one. Isaac Barrow, who in turn was familiar with the works of Bonaventura Cavalieri, probably influenced Newton. Cavalieri thought of the tangent at a point on a curve as the direction which a particle was following while at that point. The word *fluxion* is derived from the Latin word *fluxus* which means flowing. Newton imagined a particle having two components of velocity – one parallel to the x-axis and another parallel to the y-axis tracing a curve in the xy-plane. He denoted these velocities $\dot{x}$and $\dot{y}$, and called them the *fluxion* of x and the *fluxion* of y, respectively. Note that Newton involved time in his calculus. He called x and y the *fluent* of $\dot{x}$and the *fluent* of $\dot{y}$, respectively. In other words, the fluxion is the inverse of the fluent and vice versa. Then he introduced the letter o to signify an infinitely small (or *infinitesimal*) interval of time. Hence, in the case $y = x^n$, in an interval of time o, x becomes $x + \dot{x}\,o$ and y becomes $y + \dot{y}\,o$. Newton discovered the binomial theorem and so he had no problem in expanding $(x + \dot{x}\,o)^n$. Then he eliminated the terms without o (namely, y and x^n) and divided throughout by o. Since o is considered as infinitely small, we then neglect terms with o, o^2 and higher powers and obtain $\dot{y} = nx^n\ \dot{x}$. Newton was uncertain as regards the last step, which he referred to as "blotting out the o's". He admitted that his method is "shortly explained rather than accurately demonstrated." Newton relied heavily on intuition. However he got close, as the following extract from the *Principia* (Vol I, Sect. I, Lemma I.) shows:

Quantities, and the ratio of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer the one to the other than by any given difference, become ultimately equal.

We have here the concept of limit presented in such a way that led to confusion and to criticism. However, Newton emphasised that fluxions are never considered alone but in ratios.

The fact that o, initially non-zero and then practically set to zero led George Berkeley, an Irish dean in the Church of England, to criticise Newton and called infinitesimals "the ghosts of departed quantities" in his book *The Analyst*. Thus, concluded Berkeley, "he who can digest a second or third fluxions ... need not, me thinks, be squeamish to accept anything in divinity" since both led to true results.

Leibniz employed *differentials*, that is differences, and hence the letter d in the calculus. He did not involve time at all. His approach was geometrical and is in fact the one we now employ. The change in variable x was denoted by dx. In Newton's notation this is equivalent to $\dot{x}o$. He called the quantity dx the *differential of x*. His analysis was based on the *differences between the coordinates* of two neighbouring points. Note that as yet there is no notion of a derivative. Like Newton, he neglected terms with products of infinitesimals. He justified this by appealing to intuition. In his 1684 paper he gave, correctly, various results including the product and quotient rule, without any derivations.

Both Leibniz and Newton had problems in explaining away the disappearance or neglect of terms involving products of infinitesimals. Such explanations had to wait a hundred years. Leibniz appealed to the then philosophical concept of continuity to justify the limit when the infinitesimals become zero. Nowadays we define continuity by means of limits and not vice-versa. Newton was no more nearer to an adequate explanation. Because of such problems the calculus was not at first universally accepted. Christian Huygens did not accept it but he did not attack it.

Another problem, which beset Leibniz, was the notion of higher differentials. Leibniz did not regard the ratio $\frac{dy}{dx}$, which is equivalent to $\frac{\dot{y}}{\dot{x}}$ in terms of Newton's fluxions, as fundamental, and hence he could not give a satisfactory definition of d^2y. Leibniz appealed to an analogy: if we picture motion as a line, then the velocity is represented by an infinitely smaller line, and the acceleration by a doubly infinite smaller line. In a letter from Johann Bernoulli to Leibniz in which expressions such as $\sqrt[3]{d^6y} = d^2y$ are used liberally, shows the extent of the lack of clarity in the concepts.

The study of the convergence of series, essential to the idea of limit, which in turn is essential in both the differential and integral calculus, was lacking. Leibniz, in fact, seriously considered whether the series $1 - 1 + 1 - 1 + \ldots$ converged to ½. Having proved that $d(x^n) = nx^{n-1}dx$ for integral values of n he then assumed the result for rational values of n. Newton expanded $(x + o)^n$ for fractional values of n as an infinite series. He assumed convergence without any proof. Leibniz thought of $\frac{dy}{dx}$, as a ratio rather than as a limit.

It is unfortunate that both men lacked rigour. But despite this there was no lack of results This was especially true on the continent, where Leibniz method was used with advantage in preference to fluxions. The Bernoulli's produced many useful results.

Newton and Leibniz differed from others before them in that their methods were general. As stated above, rigorous proofs were lacking. Newton and Leibniz were the first to recognize the relation between the problem of tangents and that of quadratures (areas) – that one was the inverse of the other. This is the fundamental theorem of the calculus. However, we must not forget that others before Newton and Leibniz did important work concerning tangents and areas. John Barrow, and Fermat were close to the discovery of the calculus. Thus Newton and Leibniz "stood on the shoulders of giants".

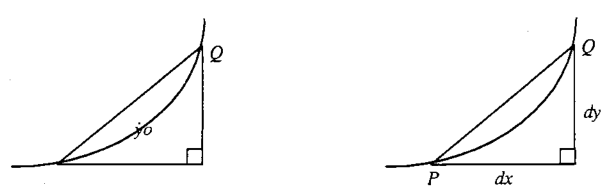

Fig. 1 *Newton's fluxions and Leibniz' differentials compared.*

Before the Calculus

We shall first examine briefly the work Fermat and Barrow, two figures who influenced both Newton and Leibniz. The idea of changing the independent variable of a function by a small amount and then setting that amount equal to zero was not something new at the time of Newton and Leibniz. Fermat employed it in his method for finding maxima. To find the maximum area of a rectangle with sides x and $(a - x)$ with $x < a$, he altered x by a small amount E. Fermat then argued that near the maximum the areas of both rectangles should be nearly equal, and by neglecting E, he obtained correctly $x = a/2$. It may be said that infinitesimals were accepted after Fermat applied them with success to this and other problems.

His method of finding tangents is illustrated in Fig. 2. We first find the subtangent TQ as follows. Let $OT = a$, $OQ = x$ and $QQ¢ = E$. We note in passing that E is Newton's $\dot{x}\,o$ and Leibniz' dx. We shall illustrate this method for the parabola $y = x^2$. We note that for nearby points P and P' we have $QP = x^2$ and $Q'P¢' < (x + E)^2$. Hence we have

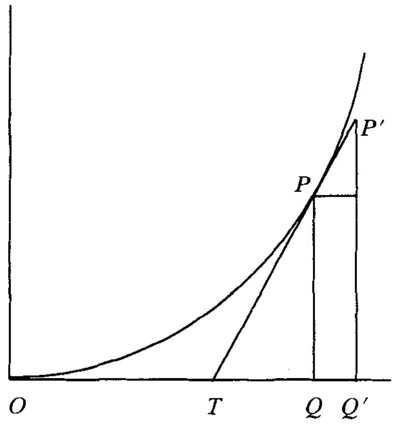

Fig.2 *Fermat's diagram for his method of finding tangents*

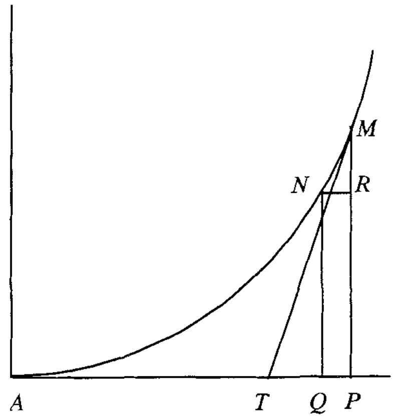

Fig.3 *Barrow's diagram for his method of finding tangents*

$$\frac{(x+E)^2}{TQ'} > \frac{P'Q'}{TQ'} = \frac{PQ}{TQ} \Rightarrow \frac{(x+E)^2}{x+E-a} > \frac{x^2}{x-a}$$

After cross-multiplying, opening brackets and simplifying, we obtain

$2x(x-a)E + E^2(x-a) = x^2E$

After dividing by E and simplifying we end with

$x^2 + xE - aE = 2xa.$

Fermat then argued that equality holds if we set $E = 0$ and hence we obtain $a = ½\,x$. Hence the tangent to the curve at P is

$$\frac{PQ}{TQ} = \frac{x^2}{x-a} = \frac{2x^2}{x} = 2x$$

Note that the idea of a limit does not occur in the argument. Fermat did not have the binomial theorem, and hence he failed to obtain general results. Barrow's method (see Fig. 3) was essentially similar to that of Fermat. But the rules he gave were more general. They could be applied for implicit equations. He laid down the following rules:

1. Let $MR = a$ and $NR = e$;
2. Substitute the values $x - e$ and $y - a$ for x and y, respectively, in the equation;
3. Reject those terms with powers of a and e or terms with ae, etc.;
4. Equate the known terms (i.e. terms without a or e) to zero;
5. Substitute MP for a and TP for e, and hence determine TP.

Here, of course, in stating (5) Barrow makes the assumption that the points M and N are close. The idea of a limit, although not expressed, is inherent in this assumption. The letters a and e are our more familiar Δy and Δx, respectively. Barrow did not mention Fermat, so we may perhaps assume that he was not familiar with Fermat's method.

We now consider the Fermat's method of quadrature for the equation $y = x^{p/q}$. Consider the points on the x-axis $x, ex, e^2x, e^3x, \ldots$, where $e < 1$. We have thus a set of ever-diminishing intervals. If we construct rectangles from these points, as shown in Fig. 4, we can approximate the area under the curve by the sum of terms of the form

$$(e^{n-1}x)^{\frac{p}{q}} \cdot \left(e^{n-1} - e^{n}\right)x = x^{\frac{p+q}{q}} e^{\frac{p+q}{q}(n-1)}(1-e)$$

Hence, summing to infinity starting from $n = 1$, we obtain

$$x^{\frac{p+q}{q}}(1-e)\Big/\left[1-e^{\frac{p+q}{q}}\right]$$

The nearer e is to 1 the more accurate will this expression for the area be. Before doing this Fermat made the substitution $e = E^q$. Noting that
$1 - e = 1 - E^q = (1-E)(1 + E + E^2 + \ldots + E^{q-1})$
and that

$1 - e^{\frac{p+q}{q}} = 1 - E^{p+q} = (1-E)(1 + E + E^2 + \ldots + E^{p+q-1})$
we substitute this in the equation. Setting $E = 1$, we obtain the area $\frac{p}{p+q}\, x^{\frac{p+q}{q}}$.

One is almost tempted to attribute the discovery of the calculus to Fermat. But one must bear in mind that Fermat saw no connection between the problem of tangents and that of quadrature. Furthermore, he did not recognise "differentiation" and "integration" as operators in themselves independent of geometrical applications.

The dispute erupts

In 1711 the Secretary of the Royal Society received a letter from Leibniz, who was also a member. Leibniz complained that in a paper to the *Philosophical Transactions*, the author Dr. John Keill insulted him. In

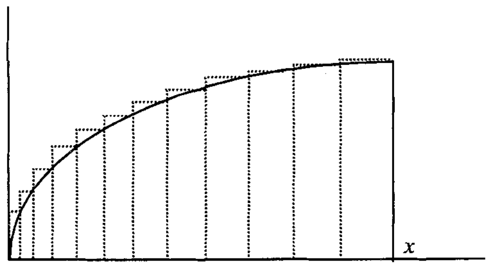

Fig. 4 *Fermat's method of quadrature.*

this paper Keill gave Newton precedence in the discovery of the calculus. Newton saw Leibniz' letter as a warning – his priority in the discovery was at stake. Newton chose his champions. Foremost among these were Keill himself, the Secretary Hans Sloane, Roger Cotes, and Edmund Halley. In 1712 the Society set up "a numerous Committee of Gentlemen of Several Nations" whose aim was to investigate Leibniz' accusation. The nations represented in this, ostensibly, impartial committee were England, Scotland and Ireland. A Prussian ambassador and a Huguenot émigré were also thrown in.

In 1713 all the relevant correspondence between Newton and Collins was published in a volume known as *Commercium epistolicum.* The outcome from this committee the following year was as expected. Keill was exonerated by the report and Leibniz was accused of plagiarism.

Both Newton and Leibniz worked behind the scenes. In 1713 an anonymous printed sheet known as the *Charta volans* spread quickly throughout the continent. This paper emphasised the fact that Newton published nothing about the calculus before Leibniz. It quoted a "leading mathematician" who said that in 1670 Newton invented only his method of infinite series. The "leading mathematician" was Johann Bernoulli, who asked Leibniz not to mention him. Johann and his nephew Nikolaus were prominent, albeit reluctant, allies of Leibniz throughout the dispute. They were not aware that Leibniz saw the *De analysi* when he was in London.

In the same year the inaugural issue of *Journal Literaire* carried an anonymous letter by Keill in defence of Newton. A French translation of the report by the Royal Society was included. Leibniz responded in the same journal by publishing (anonymously) a translation of the *Charta Volans* and a treatise on the difference between Newton's and Leibniz' methods. The author argued that in the *Principia* Newton did not make use of the calculus.

The response was another communication by Keill in the same journal published in 1714. The January and February issue of the *Philosophical Transactions of the Royal Society* was devoted, except for three pages, to Newton's cause. The dispute was, of course, officially between Keill and Leibniz.

In 1716 a number of foreign ambassadors assembled at the Royal Society to examine the documents which comprised the *Commercium epistolicum.* They recommended that Newton and Leibniz should communicate directly. Newton was thus compelled to answer Leibniz' letter. In his reply to Leibniz' first letter he called the "leading mathematician" quoted in the *Charta volans* as a "pretended mathematician". Leibniz showed the letter to Bernoulli in an attempt to provoke him. The correspondence lasted five rounds of letters, the length of which increased with every round. The correspondence came to an end with Leibniz' death in December.

The matter did not end with Leibniz' death. Six years after his death The Royal Society published a review, the *Recensio*, in which the dispute was recapitulated in Newton's favour. Needless to say the author was Newton. In 1722 the *Commercium epistolicum* was republished with revisions and footnotes that were unannounced in any preface.

Newton had his enemies on English soil. Dr. John Woodward informed Leibniz that whatever was done against him "proceeded solely from Mr. Newton" and hoped that he would not blame the Royal Society. He also promised to get him a copy of the *Commercium epistolicum.* The Astronomer Royal, John Flamsteed, who at the time of the dispute was immersed in one himself with Newton, sent Leibniz a list of mistakes in Newton's lunar theory.

Consequences

Newton's victory in the dispute was, in a sense, unfortunate. His method of fluxions was cumbersome. His notation and method were still in use in England in 1816. There was confusion between fluxions and infinitesimals as is apparent in Joseph Raphson's book *The History of Fluxions* published in 1715. He unjustly criticised Leibniz method and notation as "less apt and laborious" and as a "far-fetched symbolizing" and "insignificant novelties". Meanwhile, Leibniz' notation and method were adopted with great success on the continent.

Just as a clear language is essential to good literature, a clear mathematical notation is essential to the development of mathematics. The Newton-Leibniz dispute shows us that mathematicians may allow politics to influence their adoptions and impair their judgement. When the calculus was put on a firm footing in the 19th century, Leibniz notation was universally adopted. All that remains of Newton's notation is as the derivative of x with respect to time. But no modern teacher refers to as the fluxion of x.

The dispute occurred at a time when scientific societies under royal patronage came into existence. An ambitious man accepted and honoured by members of an established society meant instant fame which in turn guaranteed employment. His papers would be published and read widely. These societies had a controlling influence on intellectual life – they could make or break an aspiring intellectual. When Newton established himself as President of the Royal Society he became arrogant and autocratic. Newton controlled the membership and even cut short debates. We have already seen how Newton used the society for his own ends in his dispute with Leibniz.

It appears that Newton was more interested in forming a "Newtonian" school rather than furthering the advancement of science. He failed to recognise the superiority of Leibniz method over his own. Both Newton and Leibniz unashamedly used journals to further their own cause. However, it must be acknowledged that the *Acta eruditorum*, unlike the *Philosophical Transactions*, published papers from both sides of the divide. Although printing was advanced enough for the spread of knowledge, yet communication between individuals was still precarious. No postal system existed and one had to find travellers to act as couriers. When Oldenburg died communication between Newton and Leibniz ceased.

One cannot end this section without commenting on Newton's reluctance to publish. One of the reasons he avoided publishing was to avoid controversy. When he published a paper on corpuscular theory of light, it was challenged by the adherents of the wave theory. This controversy (not by any means a dispute) occupied him for some time and distracted him from his other pursuits, mainly alchemy. Newton, unlike Leibniz, stood to lose only fame by not publishing. Leibniz, on the other hand, was not a man of means. Fame meant employment, which is essential for financial security. Hence his unwillingness to share the discovery of the calculus with others. Newton, on the other hand, felt that his reputation was at stake. Newton could not take an accusation of plagiary lightly. In a letter to Bernoulli, after accepting his denial of attacking him personally, claimed that though he never sought fame among foreign nations, yet he had to preserve his character.

As we saw, others before Newton and Leibniz were close to the discovery of the Calculus. The time was ripe for its discovery. It is not inconceivable that Leibniz discovered the Calculus independently. In fact we now credit him with the independent, albeit later, discovery of the Calculus. There comes a time when a major theory simply begs discovery. At such times more than one individual may qualify as a discoverer.

Two deaths

Newton devoted a considerable time in his old age revising his scientific works for publication as new editions. His *Opticks* and *Principia* were bestsellers. The controversy with Leibniz did not diminish his stature or acceptance of his scientific works on the continent. Newtonianism took the known world by storm.

Newton pursued his theological researches as assiduously as his other interests. It is estimated that he wrote more than a million words on the subject. He wrote books on chronology, prophesies and on superstitious nonsense like the Cabala and Numerology. He believed in Hermes Trimegistus, a mythical ancient figure who passed on to Mankind scientific knowledge. He accepted 4004 BC as the year of creation, a date calculated by James Ussher, later Bishop of Armagh. He also gives the date of the Argonauts' expedition!

Newton died on 20th March 1727 at one o'clock in the morning. Among the pallbearers were the Lord Chancellor, and members of the Committee of the Royal Society. He was interred in Westminster Abbey.

The last years of Leibniz' life were dedicated mostly to the dispute. But he still remained creative and in 1710 he published *Théodicée*, a philosophical work in which he tackled the problem of good and evil. In 1714 he published *Monadologia*, perhaps his most influential philosophical work. On 14th November 1716 he died in Hanover after long suffering from arthritis and gout. Only his secretary, Eckhart, attended his funeral.

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[1] For the Leibniz biography I have relied heavily on the Internet site http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/Leibniz.html

[2] William Jones introduced the symbol π for the ratio of the circumference to the diameter of a circle.

[3] Manuel, F. E. 1968, p.260

[4] For this section I have relied heavily on Chapter 5 of Boyer, C.B., 1959.