



Research Article

Block Decomposition of Price Multipliers in a SAM Framework for Malta

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Abstract. A social accounting matrix can be utilised to model prices, providing a deeper understanding on price effects linked to intersectoral linkages, wages costs and cost-of-living adjustments. The objective of this article is to estimate price multipliers and their decomposition effects for the Maltese economy, based on the 2010 micro social accounting matrix. The estimated price multipliers follow the methodological framework proposed by Roland-Holst et al. (1995). The aim is to capture all existing price multiplier effects which are embedded within the entire circular flow of income and expenditure. This paper presents the first block decomposition of price multipliers locally with the objective to estimate and distinguish between transfer, open-loop and closed-loop effects. Therefore, this paper provides additional insight on tracing the different price effects following exogenous cost injections. Findings portray that the effects on production activities following injection in the same production account is dominated mainly by transfer effects. However, the price multiplier effects on endogenous accounts following an injection in production activities would result in mainly open-loop effects. The effects of higher wage costs on production activities and households are mainly dominated by open-loop effects, followed by induced effects. The estimated price multipliers can be utilised for policy formulation but are subject to the traditional input-output framework assumptions. However, the estimated price multipliers provide a first cut estimate of assessing price effects in terms of intermediary input costs, wages costs and cost-of-living adjustments following exogenous cost changes.

Keywords: Price Multipliers, Price Decomposition Effects, SAM-Price Model, Social Accounting Matrix.

1 Introduction

The objective of this study is to estimate the price multipliers for the Maltese economy based on the SAM framework. For the context of Malta, this is the first study that attempts to estimate price multipliers based on a Social Accounting Matrix (SAM). The estimated price multipliers follow the methodological framework proposed by Roland-Holst et al. (1995). Within the local context there had already been a study that introduced (Cassar, 2013) and applied (Theuma, 2018) the Leontief Price model. In the undergraduate dissertation by Theuma (2018), the author utilises a Symmetric input output table (SIOT) as the basis to simulate inter-sectoral price changes within the Maltese economy via the Leontief Price model¹. The price effects coming from the additional production generated throughout the economy following the increased income that is originally triggered from greater household demand were not captured in Theuma (2018) since a SIOT was utilised for the basis of the model rather than a SAM. These missing links are referred to as closed-loop or induced effects.

Two years after the study by Theuma (2018), a 2010 Maltese SAM was constructed in the post-graduate dissertation by Theuma (2020), which shall be utilised as the basis for this paper. It was the study by Cassar (2013) that constructed the first reliable and coherent SAM for Malta for the year 2000. The SAM endogenising property allows price multipliers to take into account the closed-loop effects. Therefore, the results presented in this study extend the analysis to capture the price effects originating from the extra production coming from the additional income generated as a result of higher employment to satisfy the greater household demand. In terms of prices,

¹In this study, prices refer to commodity prices, wage costs or cost-of-living effects.

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the multipliers estimated in this study capture the relationship between production activities, factors and institutions that cover the complete circular flow of income and expenditure of the Maltese economy. The estimated price multipliers effects are subject to exogenous shocks to any factor of primary inputs. Primary inputs consist of Gross Value Added (GVA), imports and net taxes on products. Also, GVA consists of compensation of employees, net operating turnover, sales of fixed capital and net taxes on production. Therefore, price fluctuations can be assessed with the estimated price multipliers following exogenous changes in any element of primary inputs.

A Symmetric Input-Output Table (SIOT) shows sectoral information regarding the production and consumption of an economy. The SIOT framework can be expanded to endogenise household behaviour, but it does not reflect the entire circular flow of income and expenditure of an economy. Models with households assumed as endogenous, such as the SAM, allow for the estimation of multipliers that capture the entire circular flow of income and expenditure (Miller et al., 2009). Therefore, a SIOT does not fully succeed in capturing the income effects distributed across institutions and factors. In this study, to capture the entire circular flow of income and expenditure a SAM for Malta is instead utilised. In matrix form, a SAM records the income generated by production activities, and the income distribution and redistribution between the various institutions within an economy (Round, 2003).

A SAM is also referred to as an economy-wide data framework (El-Said et al., 2001), which is considered as “comprehensive” (El-Said et al., 2001) and “flexible” (Round, 2003). The SAM framework retains a high degree of consistency because total receipts and payments throughout the economy balance. Within the SAM framework, receipts are displayed as row elements, while payments are portrayed as column elements. The flexibility of the SAM structure allows its accounts to be disaggregated according to the scope of study. A SAM can have different levels of disaggregation, such that economic activities can be analysed at a macro or micro level. A SAM with a single aggregated production account is referred to as a Macro SAM, which is mainly used to analyse an economy at a macro level, hence its name. On the other hand, when the production activities, factors and final demand are disaggregated at a sectoral level, it is generally referred to as a Micro SAM. As its name suggests, a Micro SAM is generally utilised to analyse the economy at a sectoral level. Other different SAM types also exist, such as household, financial or environmental-extended SAMs, which further disaggregate the Macro or Micro SAM accounts. The disaggregation level of the

SAM accounts vary depending on the scope of study.

The next section provides an in-depth explanation of the methodology utilised to estimate the price multipliers and their decomposition. The Methodology Section starts off with a brief description of the data sources utilised to obtain the Micro SAM utilised for this study. It is then followed by an explanation on splitting the SAM accounts between endogenous and exogenous, which is later utilised as the basis to estimate and decompose the price multipliers for every Maltese economic sector. Two block-decomposition techniques shall be carried out, the multiplicative and the additive methods. The results are then put forward and discussed in section 3. The paper concludes with a summary of the main findings.

2 Data and Methodology

A SAM shall be utilised as the basis to estimate price multipliers and their decomposition effects to capture the entire circular flow of income and expenditure. In the postgraduate dissertation by Theuma (2020), the author had constructed a 2010 Micro SAM for Malta utilising reliable official statistical data sources. The Micro SAM also conforms to the latest European System of Accounts (Commission et al., 2013) framework and NACE (Eurostat, 2008) classification. The 2010 Maltese Micro SAM was readily available, disaggregated at a 44 sectoral level, with two factor accounts (labour and capital) and five institution accounts, of which three are considered as domestic, namely (i) Households, (ii) Government and (iii) Enterprises.

Subject to the traditional input-output framework and additional assumptions imposed during the SAM construction process², the comprehensive economy-wide data framework can be utilised to undertake multiplier analysis. In other words, the consequent effects of exogenous injections in an economy can be studied via multiplier or impact analysis. However, this requires a SAM to be partitioned into endogenous and exogenous accounts as shown in table 1. For consistency purposes, the same methodological structure put forward in Roland-Holst et al. (1995) is utilised. To capture the closed-loop or induced effects and obtain a complete set of price multipliers, production activities, factors, and households and enterprises institutions are taken as endogenous as portrayed in table 1. Government, taxes, capital and Rest of World accounts are taken as exogenous. The endogenous and exogenous partitioning structure adopted in this study also conforms to that adopted by Defourny et al. (1984), Pyatt et al. (1979) and Roland-Holst et al. (1995).

²Refer to Theuma (2020) for a detailed description of the additional assumptions imposed during the 2010 Maltese SAM construction process

It is worth noting that for the constructed 2010 Maltese SAM, the flows between the production activities and the enterprises institution are directly included under the production account rather than found separately within the enterprises institution column (Theuma, 2020). Therefore, payments by enterprises to production activities are assumed to be zero to avoid double counting. Also, the domestic household institution account also includes flows of Non-Profit Institutions Serving Households (NPISH).

From table 1, the flows amongst all endogenous accounts (\mathbf{T}_{11} , \mathbf{T}_{13} , \mathbf{T}_{21} , \mathbf{T}_{32} , \mathbf{T}_{33}) are portrayed as five separate block matrices, grouped into a single square (3×3) matrix. Block matrix \mathbf{T}_{11} mirrors the intermediate consumption matrix that is readily found within a SIOT. Block matrix \mathbf{T}_{13} denotes the expenditure of the endogenised domestic institutions on total output. Block matrix \mathbf{T}_{21} denotes the value-added generated from factors by all production activities. Block matrix \mathbf{T}_{32} denotes the income received by the endogenous domestic institutions for their labour services. Block matrix \mathbf{T}_{33} denotes the flows between the endogenised domestic institutions. Block matrix \mathbf{T}_{14} denotes final demand for all the exogenous accounts, which is identical to the final demand column found within a SIOT. Block matrix \mathbf{T}_{24} denotes final demand for factors of production by exogenous accounts, which is required for output production. Block matrix \mathbf{T}_{34} denotes the income received by all endogenised domestic institutions from all exogenous institutions. Similar to the injection column vector (\mathbf{T}_{14} , \mathbf{T}_{24} , \mathbf{T}_{34}), the leakages row vector (\mathbf{T}_{41} , \mathbf{T}_{42} , \mathbf{T}_{43}) denotes the flows from endogenous to exogenous accounts in the form imports, savings, and taxes, respectively. Block matrix \mathbf{T}_{44} represents the inter-institutional transactions between all exogenous institutions. Total receipts and payments in table 1 must equate, such that every column element (\mathbf{Y}_1 , \mathbf{Y}_2 , \mathbf{Y}_3 , \mathbf{Y}_4) and row element (\mathbf{Y}_1 , \mathbf{Y}_2 , \mathbf{Y}_3 , \mathbf{Y}_4) within the SAM structure balance (Theuma, 2020). table 1 can also be portrayed in such a way to provide a picture of the entire circular flow of income and expenditure of an economy, as displayed in figure 1.

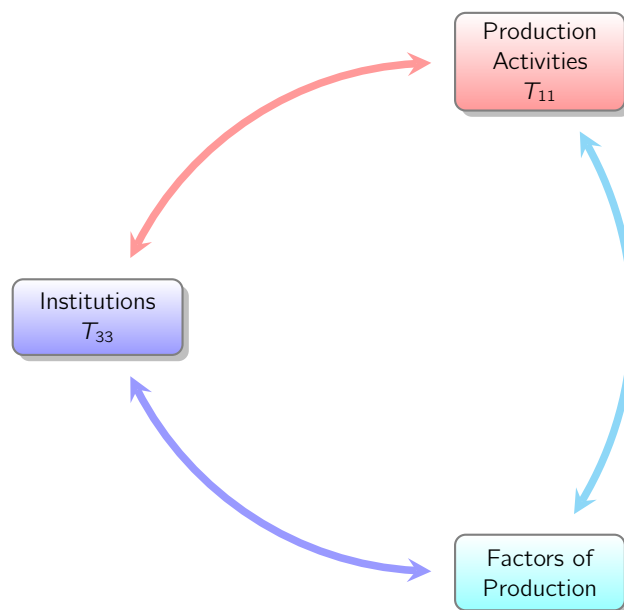


Figure 1: Circular Flow of Income and Expenditure

Source: (Defourny et al., 1984).

From figure 1, departing from block matrix \mathbf{T}_{11} , income is generated from the value-added taking place during the production processes of economic sectors. The newly generated income is then transformed into factors of production, which are supplied by the various institutions present in the economic system to produce goods and services. This process is visualised as \mathbf{T}_{21} in table 1. Factors of production are then accrued to institutions, representing amongst others, the income distribution to households. Institutions can have transactions between themselves, \mathbf{T}_{33} , such as inter-income transfers between households. The income generated and distributed to households is then utilised for expenditure of goods and services by the various economic institutions \mathbf{T}_{13} as portrayed in table 1. The cycle repeats itself portraying the movements and generation of income from producers to workers and back.

Utilising the partitioning structure of table 1, we can define matrix \mathbf{A} that denotes the average expenditure propensities, representing the fixed interactions between the inputs and output of every sector. These sectoral fixed interactions are generally denoted as \mathbf{a}_{ij} , portraying the input \mathbf{i} required by sector \mathbf{j} to produce a unit of a commodity. Therefore, $\mathbf{a}_{ij}\mathbf{y}_j$ are required to produce \mathbf{t}_{ij} number of goods. From table 1, in matrix algebra notation \mathbf{a}_{ij} can be made subject of the formula to represent the technical coefficients matrix.

Partitions	Endogenous	Endogenous	Endogenous	Exogenous	Total	
	SAM Accounts	Production Activities	Factors	Household and Enterprises	Government, Taxes, Capital and RoW	
Endogenous	Production Activities	T_{11}	0	T_{13}	T_{14}	Y_1
Endogenous	Factors	T_{21}	0	0	T_{24}	Y_2
Endogenous	Household and Enterprises	0	T_{32}	T_{33}	T_{34}	Y_3
Exogenous	Government, Taxes, Capital and RoW	T_{41}	T_{42}	T_{43}	T_{44}	Y_4
Total		Y_1	Y_2	Y_3	Y_4	

Table 1: SAM Framework's Schematic Structure

Source: (Roland-Holst et al., 1995).

$$\mathbf{T} = \mathbf{A}\hat{\mathbf{Y}} \tag{1}$$

$$\hat{\mathbf{T}}\mathbf{Y}^{-1} = \mathbf{A} \tag{2}$$

$$\frac{t_{ij}}{y_j} = a_{ij} \tag{3}$$

Where square matrix \mathbf{A} is made up of only endogenous SAM accounts, square matrix \mathbf{T} denotes the expenditure by the endogenous accounts and matrix $\hat{\mathbf{Y}}$ denotes a square matrix with diagonal elements \mathbf{y}_i ($i = 1, \dots, n$). The matrix of average expenditure propensities \mathbf{A} utilised in this paper is an augmented version to that derived from a SIOT, since it now includes all flows of the endogenous accounts. Since matrix \mathbf{A} is derived by dividing its internal five block matrices with their respective income total, it captures all the relationships between the endogenous accounts.

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{bmatrix} \tag{4}$$

Block matrix \mathbf{A}_{11} denotes the share of intermediary inputs utilised during the production processes to produce output. Block matrix \mathbf{A}_{21} denotes the share of factors required to produce a one-euro worth of output. Block

matrix \mathbf{A}_{13} denotes the mean purchases of every commodity supplied by the endogenous institutions for every one-euro worth of total expenditure. Block matrix \mathbf{A}_{32} denotes the income received by the endogenous institutions for every one-euro worth of labour services carried out. Block matrix \mathbf{A}_{33} denotes on average every one-euro worth of transfers between every endogenous institution (Cassar, 2013).

Following the endogenous and exogenous partitioning in table 1 and the standard Leontief demand-driven assumptions, the SAM-quantity model is derived next. In the SAM-quantity model, production is assumed to change while prices are assumed to remain unchanged. The justification of these assumptions come only in the presence of excess capacity and unlimited supply of resources (Roland-Holst et al., 1995). Utilising the partitioning structure derived in table 1 and Matrix \mathbf{A} of average expenditure propensities, the SAM-quantity model can be utilised to derive income of group 1 by:

$$\mathbf{Y}_1 = \mathbf{A}_{11}\mathbf{Y}_1 + \mathbf{A}_{13}\bar{\mathbf{Y}}_3 + \mathbf{A}_{14}\bar{\mathbf{Y}}_4 \tag{5}$$

$$= (\mathbf{I} - \mathbf{A}_{11})^{-1}(\mathbf{A}_{13}\bar{\mathbf{Y}}_3 + \mathbf{A}_{14}\bar{\mathbf{Y}}_4) \tag{6}$$

$$= \mathbf{M}_{11}\mathbf{x} \tag{7}$$

Where $\mathbf{M}_{11} = (\mathbf{I} - \mathbf{A}_{11})^{-1}$ denotes the SAM Leontief inverse matrix that mirrors the SIOT Leontief inverse or

multiplier matrix, matrix \mathbf{A}_{11} mirrors the SIOT Leontief technical coefficients matrix \mathbf{A} , and \mathbf{x} mirrors the SIOT exogenous final demand column vector for every economic sector. To distinguish between endogenous and exogenous elements, those elements with a bar represent the SAM accounts partitioned as exogenous, while those without represent the remaining endogenous accounts. Therefore, the consequent effects on endogenous production following a one euro change in exogenous activity can be assessed. This can also be expressed as $\Delta \mathbf{Y}_1 = \mathbf{M}_{11} \Delta \mathbf{x}$.

The SAM-Price model, which is dual to the SAM-quantity model can also be explored. The assumptions change, whereby prices are allowed to change, while quantities are assumed to remain fixed. Let \mathbf{p}_i represent a price index for the production activities of group i . Utilising the same endogenous and exogenous split between the SAM accounts denoted in table 1, the first column can be represented in matrix algebra form as:

$$\mathbf{p}'_1 = \mathbf{p}'_1 \mathbf{A}_{11} + \bar{\mathbf{p}}'_2 \mathbf{A}_{21} + \bar{\mathbf{p}}'_4 \mathbf{A}_{41} \quad (8)$$

$$= (\bar{\mathbf{p}}'_2 \mathbf{A}_{21} + \bar{\mathbf{p}}'_4 \mathbf{A}_{41}) (\mathbf{I} - \mathbf{A}_{11})^{-1} \quad (9)$$

$$= \mathbf{v}'_1 \mathbf{M}_{11} \quad (10)$$

Where \mathbf{v}'_1 denotes a row vector of exogenous costs, and \mathbf{M}_{11} denotes the same Leontief inverse matrix found in the Leontief Demand-Driven model balance equation. Dual to the SAM-quantity model, the SAM-price model can be utilised to assess the effects of a one euro exogenous cost change on prices $\Delta \mathbf{p}'_1 = \Delta \mathbf{v}'_1 \mathbf{M}_{11}$. Utilising table 1, the above can be expanded to represent a set of linear equation in terms of prices.

$$\mathbf{p}'_1 = \mathbf{p}'_1 \mathbf{A}_{11} + \mathbf{p}'_2 \mathbf{A}_{21} + \bar{\mathbf{p}}'_4 \mathbf{A}_{41} \quad (11)$$

$$\mathbf{p}'_2 = \mathbf{p}'_3 \mathbf{A}_{32} + \bar{\mathbf{p}}'_4 \mathbf{A}_{42} \quad (12)$$

$$\mathbf{p}'_3 = \mathbf{p}'_1 \mathbf{A}_{13} + \mathbf{p}'_3 \mathbf{A}_{33} + \bar{\mathbf{p}}'_4 \mathbf{A}_{43} \quad (13)$$

Let \mathbf{p}' denote a row vector of prices ($\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}'_3$) for the endogenous SAM accounts and $\mathbf{v}' = \mathbf{p}'_4 \mathbf{A}_4$ denotes the row vector of exogenous costs, where \mathbf{A}_4 comprises of row elements ($\mathbf{A}_{41}, \mathbf{A}_{42}, \mathbf{A}_{43}$). The balance equation of the SAM-Price model is summarised as:

$$\mathbf{p}' = \mathbf{v}' (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{v}' \mathbf{M} \quad (14)$$

Where \mathbf{p}' is a row vector of unitary prices, \mathbf{v}' is a row vector portraying the ratio of primary inputs expenditure required by each sector to produce one monetary unit worth of output ($\frac{v_i}{y_n}$) and \mathbf{M} is a multiplier matrix which is common for the Leontief quantity and price models. However, the interpretation of the multiplier matrix is different

between the two dual models. For the interpretation of the Leontief SAM-price model, the rows across matrix \mathbf{M} are interpreted. For the Leontief SAM-quantity model or the SAM-based output (production) multipliers³, the columns of matrix \mathbf{M} are instead interpreted. To distinguish between the two possible interpretations of the multiplier matrix \mathbf{M} , its transpose \mathbf{M}' will be referred to as the price transmission matrix.

Stone (1936) and Pyatt et al. (1979) introduce the multiplicative and additive block-decomposition methods to disaggregate matrix \mathbf{M} or \mathbf{M}' into three separate block matrices which have important economic meaning⁴. The decomposition of the multiplier matrix allows for the interpretation of the (i) transfers, (ii) open-loop, and (iii) closed-loop matrices. The three respective block-matrices reflect the consequent effects on the endogenous partitioned group as shown in table 1 following an exogenous injection in the economic system. The multiplicative block-decomposition method shall be primarily carried out to estimate the three decomposition multiplier matrices ($\mathbf{M1}, \mathbf{M2}, \mathbf{M3}$). The additive block-decomposition method is applied afterwards to be able and provide a clear interpretation of the three different price multiplier effects. The multiplicative block-decomposition method is presented next, which starts off from the SAM-Price model and the introduction of a new matrix $\tilde{\mathbf{A}}$.

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{33} \end{bmatrix} \quad (15)$$

Matrix $\tilde{\mathbf{A}}$ is a square matrix of average expenditure propensities, extracted directly from matrix \mathbf{A} . Matrix $\tilde{\mathbf{A}}$ must satisfy the condition that $(\mathbf{I} - \tilde{\mathbf{A}})$ is invertible (Roland-Holst et al., 1995).

³For the context of this paper, the SAM-based output (production) multipliers go beyond the scope of this study. However, a more detailed analysis on the output (production) multiplier for the Maltese economy can be found in Cassar (2013, 2015), Cassar et al. (2018) and Theuma (2020).

⁴The two block-decomposition methods applied in this paper follow the methodological framework proposed by Stone (1936) and Pyatt et al. (1979).

$$\mathbf{p}' = \mathbf{v}'\mathbf{M} \quad (16)$$

$$= \mathbf{p}'\mathbf{A} + \mathbf{v}' \quad (17)$$

$$= \mathbf{p}'\mathbf{A} + \mathbf{p}'\tilde{\mathbf{A}} - \mathbf{p}'\tilde{\mathbf{A}} + \mathbf{v}' \quad (18)$$

$$= \mathbf{p}'(\mathbf{A} - \tilde{\mathbf{A}})(\mathbf{I} - \tilde{\mathbf{A}})^{-1} + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1} \quad (19)$$

$$= \mathbf{p}'\mathbf{A}^* + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1} \quad (20)$$

$$= [\mathbf{p}'\mathbf{A}^* + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}]\mathbf{A}^* + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1} \quad (21)$$

$$= \mathbf{p}'\mathbf{A}^{*2} + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}(\mathbf{I} + \mathbf{A}^*) \quad (22)$$

$$= [\mathbf{p}'\mathbf{A}^* + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}]\mathbf{A}^{*2} + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}(\mathbf{I} + \mathbf{A}^*) \quad (23)$$

$$= \mathbf{p}'\mathbf{A}^{*3} + \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}(\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2}) \quad (24)$$

$$= \mathbf{v}'(\mathbf{I} - \tilde{\mathbf{A}})^{-1}(\mathbf{I} + \mathbf{A}^* + \mathbf{A}^{*2})(\mathbf{I} - \mathbf{A}^{*3})^{-1} \quad (25)$$

$$= \mathbf{v}'\mathbf{M}_1\mathbf{M}_2\mathbf{M}_3 \quad (26)$$

Where $\mathbf{A}^* = (\mathbf{A} - \tilde{\mathbf{A}})(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$ and the multiplication of $\mathbf{M}_1\mathbf{M}_2\mathbf{M}_3$ equates to the SAM Leontief multiplier matrix \mathbf{M} . By taking the transpose of the three block-multiplier decomposition matrices, a new equation is obtained that satisfies the price transmission matrix.

$$\mathbf{p} = \mathbf{M}'\mathbf{v} = \mathbf{M}'_3\mathbf{M}'_2\mathbf{M}'_1\mathbf{v} \quad (27)$$

Where:

$$\mathbf{M}'_1 = \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{11})^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\mathbf{I} - \mathbf{A}_{33})^{-1} \end{bmatrix} \quad (28)$$

$$\mathbf{M}'_2 = \begin{bmatrix} \mathbf{I} & \mathbf{A}_{21}^* & \mathbf{A}_{32}^*\mathbf{A}_{21}^* \\ \mathbf{A}_{13}^* & \mathbf{A}_{32}^* & \mathbf{A}_{32}^* \\ \mathbf{A}_{13}^* & \mathbf{A}_{21}^*\mathbf{A}_{13}^* & \mathbf{I} \end{bmatrix} \quad (29)$$

$$\mathbf{M}'_3 = \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{13}^*\mathbf{A}_{32}^*\mathbf{A}_{21}^*)^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{A}_{21}^*\mathbf{A}_{13}^*\mathbf{A}_{32}^*)^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\mathbf{I} - \mathbf{A}_{32}^*\mathbf{A}_{21}^*\mathbf{A}_{13}^*)^{-1} \end{bmatrix} \quad (30)$$

Since the multiplier matrix \mathbf{M} has been transposed to matrix \mathbf{M}' , the interpretation of the column and row elements of the SAM multiplier matrices ($\mathbf{M}, \mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3$) have also been switched. To analyse the price multiplier effects we now look down the columns of matrix \mathbf{M}' ⁵. However, to obtain a more concise picture of the price multiplier decomposition effects, the Additive Block-Decomposition method is applied next.

$$\mathbf{M}' = \mathbf{I} + \mathbf{T} + \mathbf{O} + \mathbf{C} \quad (31)$$

$$= \mathbf{I} + (\mathbf{M}'_1 - \mathbf{I}) + (\mathbf{M}'_2 - \mathbf{I})\mathbf{M}'_1 + (\mathbf{M}'_3 - \mathbf{I})\mathbf{M}'_2\mathbf{M}'_1 \quad (32)$$

Where \mathbf{I} denotes the initial injection, $\mathbf{T} = (\mathbf{M}'_1 - \mathbf{I})$ denotes the transfer effects, $\mathbf{O} = (\mathbf{M}'_2 - \mathbf{I})\mathbf{M}'_1$ denotes the open-loop or cross multiplier effects and $\mathbf{C} = (\mathbf{M}'_3 - \mathbf{I})\mathbf{M}'_2\mathbf{M}'_1$ denotes the closed-loop or induced effects. The Additive Block-Decomposition method enables us to provide a clearer interpretation of the price multiplier decomposition estimates, which is found in the next chapter. To apply the Additive Block-Decomposition method, the three decomposed multiplier matrices are required ($\mathbf{M}'_1, \mathbf{M}'_2, \mathbf{M}'_3$). Therefore, the Multiplicative Block-Decomposition matrix is applied before the Additive Block-Decomposition method because the former method is a requirement for the latter method.

3 Results and Discussion

The block-decomposition of the price multipliers enable us to examine the various effects of price changes into more detail. As shown in table 2, the decomposed price effects are namely (i) transfer effects, (ii) open-loop effects and (iii) closed-loop effects. The transfer effects portrayed in table 2 capture the price multiplier effects originating from direct links between the endogenous accounts and inter-sectoral transactions. From example 4 of table 2, the consequent impact of exogenously shocking the production cost of the Financial Services sector by one euro would result in a direct price increase of 0.01 euro in the same sector. The open-loop effects only capture the price multiplier effects coming from transfers between every endogenous account. From example 10 of table 2, the consequent impact of exogenously shocking the production cost of the Financial Services sector by one euro would result in an increase of 0.02 euro in wage costs. The closed-loop effects, also known as induced effects, capture the price multiplier effects originating from the additional production generated in the economy that ultimately comes from greater household demand. In other words, the closed-loop effects ensure a completed circular flow of income and expenditure throughout the economy as depicted in figure 1. Therefore, price multipliers capture the price effects that can be traced back to the “big circuits of influence” (Roland-Holst et al., 1995). From table 2, when the injected (origin) and influenced (destination) sectors are the same, a one is added as an initial injection⁶ to reflect a one euro injection to the origin sector on the destination sector. Similarly, it is also possible to undertake a hypothetical scenario of any euro amount injected to any origin sector and assess the consequent price effects of any destination sector in the economy.

⁶The initial injection can also be thought of as an identity matrix with one on its diagonals embedded within the multiplier matrix \mathbf{M}'_1 .

⁵Refer to section 4 within the Appendix Chapter for Matrix \mathbf{M}' .

Example	SAM Account Injection Origin	SAM Account influenced by the injection	Initial Injection (I)	Transfer Effects (T)	Open-Loop Effects (O)	Closed-Loop Effects (C)	Price Multipliers (M_{ij}^p)
1	Accommodation and Food Services	Travel Agency	0	0.08	0	0.03	0.11
2	Accommodation and Food Services	Accommodation and Food Services	1	0.01	0	0.04	1.05
3	Financial Services	Accommodation and Food Services	0	0.08	0	0.02	0.10
4	Financial Services	Financial Services	1	0.01	0	0	1.01
5	Financial Services	Advertising and Market Research	0	0.26	0	0.02	0.27
6	Electricity, Water and Waste Services	Textiles Manufacturing	0	0.19	0	0.02	0.22
7	Electricity, Water and Waste Services	Retail Trade	0	0.10	0	0.03	0.13
8	Financial Services	Endogenous Institutions	0	0	0.03	0	0.03
9	Accommodation and Food Services	Endogenous Institutions	0	0	0.05	0.01	0.06
10	Financial Services	Factors	0	0	0.02	0	0.03
11	Accommodation and Food Services	Factors	0	0	0.05	0.01	0.06
12	Factors	Activities of Households as Employers	0	0	0.99	0.17	1.16
13	Factors	Endogenous Institutions	0	0	0.16	0.03	0.19

Table 2: 2010 Price Multiplier Decomposition Effects for Malta

Source: Authors' Own Calculations.

Table 2⁷ presents the price multipliers for domestic economic sectors, institutions, and factors. These price multipliers also include their block-decomposition effects. The Additive Block-Decomposition method is utilised to interpret and assess the results. As previously explained in the Methodology Chapter, the Multiplicative Block-Decomposition method was required to apply the Additive Block-Decomposition method. Generally, the results obtained from the Additive Block-Decomposition method are preferred for ease of interpretation. The results portrayed in table 2 were chosen to best explain the different kinds of multiplier decomposition effects that also exhibit important characteristics.

From table 2, examples 1 to 7 capture the decomposition effects of a one euro increase in exogenous costs of production activities, and the consequent influence on production prices. For instance, example 1 in table 2 portrays an approximate increase in production prices of around 0.11 euro for the Travel agency, Tour operator Reservation Service and Related Activities sector following a one euro exogenous increase in the cost of production activities originating from the Accommodation and Food Services sector. From the resulting 0.11 price multiplier increase, 0.08 euro increase takes the form of transfer ef-

fects and the remaining 0.03 euro increase take the form of closed-loop effects due to the additional production generated following the extra consumption of goods and services by households. In other words, the 0.08 euro increase in prices originates from the inter-sectoral transactions between production activities while the 0.03 euro increase in prices ultimately originates from additional household demand. In this case, the open-loop effects are zero because the origin and destination sectors belong to the same account category. In other words, the origin and destination sectors of examples 1 to 7 belong to the same production account.

From table 2, example 4 portrays that when the Financial Services sector is shocked by an initial injection (I) of one euro worth of exogenous cost, the price multiplier for that same sector amounts to 1.01 euro. Therefore, a one euro exogenous cost increase for the Financial Services sector would bring about an increase in production prices of that same sector by 1.01 euro, of which the majority is accounted by the injection itself, while a relatively small remaining effect of 0.01 euro takes the form of transfer effects. The low multiplier effects come from the Special Purpose Entities (SPEs) included in the Financial Services sector during the construction of the SIOT, which is conformant to the latest NACE Rev.2. However, since SPEs contain a substantial amount of import content, the level of leakages present in the domestic financial sector is high.

⁷In situations where the additive block-decomposition effects do not sum up to their respective price multiplier total, it is because of rounding up.

The inclusion of SPEs in the SAM reduces the magnitude of the multiplier effects attributed by the financial sector. The higher the share of leakages within the sector's production process relative to its domestic input requirements, the lower the effects of exogenous cost changes on prices would have.

Similarly, from [table 2](#), example 5 denotes a price multiplier for the Advertising sector of 0.27 euro, of which approximately 96 per cent is attributable to transfer effects. For examples 1 to 7 it can be deduced that the price multipliers are dominated by transfer effects when compared to closed-loop effects. This implies that these sectors are highly integrated but exhibit weak forward linkages (Roland-Holst et al., 1995). Therefore, these estimates shed light on the underlying cost linkages between production activities.

Examples 8 and 9 present the price multiplier effects on endogenous institutions following a one euro exogenous cost increase in production activities. From [table 2](#), example 8 portrays that the effects on the domestic households institution following a one euro exogenous cost in the Financial Services sector would result in a price multiplier effect of 0.03, which is entirely represented by open-loop effects. In example 9, a one euro exogenous injection in the Accommodation and Food Services sector would bring about a price multiplier effect of 0.06 euro on the households institution. From the 0.06 euro increase in the cost-of-living index, 0.05 takes the form of open-loop effects while the remaining 0.01 euro takes the form of closed-loop effects. Roland-Holst et al. (1995) stress that it is the tendency that for the households institution, open-loop effects dominate closed-loop effects, which is also the case for Malta as visualised in [table 2](#). Compared to Examples 1 to 7, this time the origin and destination sectors belong to a different account category. As a result, the transfer effects are now zero for examples 8 and 9.

Examples 10 and 11 denote the impact on factors following an increase in the cost of production. Consequently, Example 12 denotes the opposite linkages and captures the induced effects on production prices following exogenous cost increases in factor prices. Example 13 denotes the relationship between factors and endogenous institutions. It traces the increase in the cost of living of households following an increase in factor costs. Increasing labour costs would in turn stimulates commodity prices which ultimately push up households' cost of living. Without the application of the decomposition, it is very difficult to assess the price effects by rules of thumb between producing sectors, factors and endogenous institutions.

The SAM inherits the assumptions put forward by the

input-output framework since they are the core of the SAM. Therefore, these assumptions are important during the interpretation of the results. A detailed explanation of these assumptions can be found in Miller et al. (2009). In the case of this paper, there are particular assumptions worth mentioning. For instance, as opposed to the Leontief Demand-Driven model, the SAM-price model assumes that only prices can fluctuate in the economy, while keeping quantities fixed. Also, there is only one commodity for production in the economy. Therefore, the importance of particular commodities is not captured. This implies that there are no possibilities of substitution between inputs during production. In other words, it is also assumed that purchasers of intermediary inputs do not shift to substitute goods. In fact, commodities would be considered as perfect complements. This strong assumption might not hold in reality because sellers do switch intermediary inputs for their production process as a result of price hikes to minimise costs and safeguard profits.

Also, since the supply of resources is also assumed to be infinite, price multipliers can be utilised to determine any price level following shocks in primary inputs. However, these price levels can be estimated by imposing an important assumption that an economy has unlimited labour supply in its disposal. However, this assumption can be unrealistic because there is a limit to how much resources may be available, especially for a small island developing economy.

By assessing the price multipliers estimated in this study, it is possible to obtain a detailed insight into the price fluctuations in terms of production activities, higher wage costs and cost-of-living adjustments. Furthermore, the price decomposition estimates provide more insight on the price transmission or inter-sectoral relationship patterns in the economy in terms of prices. These estimates can also be utilised by policy makers to identify the impact on prices throughout the economy during policy design, with the aim to gain a deeper insight on the price transmission.

4 Conclusion

The objective of this paper was to estimate the price multipliers and their corresponding decomposition effects within the local context. A 2010 SAM for Malta was utilised as the basis in order to obtain a complete set of price multipliers. As a result of the endogenising feature embedded within the SAM, the price multiplier effects in the presence of endogenous and factor accounts could be appropriately assessed in detail, namely the transfer, open-loop and closed-loop price multiplier effects. In this case, the price effects on households represent simulations of cost-of-living adjustments, while the simulations on the

factor account captures the increase in factor prices due to higher wage costs.

These price multiplier estimates are subject to the traditional input-output framework assumptions. However, they also have advantages over the traditional Walrasian price models (Roland-Holst et al., 1995). For instance, a notable advantage is the ability to estimate price changes that are useful for policy making at sectoral level. In terms of prices, the decomposition of multiplier effects can also be utilised to assess the transmission or patterns of inter-sectoral relationships. The transfer, open and closed-loop multiplier effects can be assessed via the block-decomposition method, which enables us to distinguish between price effects explained by inter-sectoral linkages, consumption by households (cost-of-living) and factor prices (wage increases). In other words, the block-decomposition of price multipliers provides the possibility to assess the relationships between production, factors and institutions in terms of cost-linkages.

The estimated price multipliers can be of great importance for policy makers to assess relative price changes between production activities, factors and institutions following recent events of supply shortages, rising import costs and higher wage costs. These events may potentially lead to higher inflation rates locally (CBM, 2021). Since these factors are components of primary input costs, price changes can be assessed via the estimated price multipliers following an exogenous shock in one of these components. Therefore, the price multipliers can be utilised to assess the effects of a one euro exogenous cost change in primary inputs on relative prices. The block decomposition of price multipliers provides detailed insight on tracing the different price effects in terms of input costs, cost-of-living adjustments and wage costs.

A more in-depth analysis can be undertaken on sectoral price linkages to obtain a more comprehensive description of linkages effects on prices. This can be done via the path-decomposition method. Therefore, a further research avenue is to identify and analyse the paths through which cost effects circulate between production activities, factors and institutions throughout the economy.

References

- Cassar, I. P. (2013). *A study of the production structure of the Maltese economy: An input-output approach* (Doctoral dissertation). Heriot-Watt University.
- Cassar, I. P. (2015). *Estimates of output, income value added and employment multipliers for the Maltese economy* (tech. rep.). CBM Working Papers.
- Cassar, I. P. & Rapa, N. (2018). Estimates of input-output multipliers for the Maltese economy based on the symmetric input-output table for 2010.
- CBM. (2021). Outlook for the Maltese economy. 4.
- Commission, E. E. et al. (2013). European system of accounts ESA 2010. *Publications Office of the European Union, Luxembourg*. doi, 10, 16644.
- Defourny, J. & Thorbecke, E. (1984). Structural path analysis and multiplier decomposition within a social accounting matrix framework. *The Economic Journal*, 94(373), 111–136.
- El-Said, M., Lofgren, H. & Robinson, S. (2001). *The impact of alternative development strategies on growth and distribution: Simulations with a dynamic model for Egypt* (tech. rep.).
- Eurostat, N. (2008). Rev. 2—statistical classification of economic activities in the European community. *Office for Official Publications of the European Communities, Luxembourg*.
- Miller, R. E. & Blair, P. D. (2009). *Input-output analysis: Foundations and extensions*. Cambridge university press.
- Pyatt, G. & Round, J. I. (1979). Accounting and fixed price multipliers in a social accounting matrix framework. *The Economic Journal*, 89(356), 850–873.
- Roland-Holst, D. W. & Sancho, F. (1995). Modeling prices in a SAM structure. *The Review of Economics and Statistics*, 361–371.
- Round, J. (2003). Constructing sams for development policy analysis: Lessons learned and challenges ahead. *Economic Systems Research*, 15(2), 161–183.
- Stone, R. (1936). *Aspects of economic and social modelling* (Vol. 1). Librairie Droz.
- Theuma, A. (2018). Assessing sectoral price effect of wage increases in the Maltese economy via the leontief price model.
- Theuma, A. (2020). Contribution of the financial services and gaming sectors to the wages of Maltese households: A sam-based hea.

Appendices

Sector Number	Sector Name	NACE Rev2
1	Crop and animal production, hunting and related service activities	A01
2	Forestry and logging	A02
3	Fishing and aquaculture	A03
4	Manufacture of food products, beverages and tobacco products	C10T12
5	Manufacture of textiles, wearing apparel and leather products	C13T15
6	Manufacture of wood and of products of wood and cork	C16
7	Manufacture of paper and paper products, printing and reproduction of recorded media, manufacture of coke and refined petroleum products, chemical products, basic pharmaceutical products and pharmaceutical preparations and rubber and plastic products	C17 to C22
8	Manufacture of metallic mineral products	C23
9	Manufacture of basic metals	C24
10	Manufacture of fabricated metal products, except machinery and equipment	C25
11	Manufacture of computer, electronic and optical products, electrical equipment, motor vehicles, trailers and semi-trailers	C26 to C32
12	Repair and installation of machinery and equipment	C33
13	Electricity, gas, steam and air conditioning supply, water collection, treatment and supply, sewerage, waste collection, treatment and disposal activities, materials recovery, remediation activities and other waste management services	D35 to E39
14	Construction of buildings	F41
15	Wholesale and retail trade and repair of motor vehicles and motorcycles	G45
16	Wholesale trade, except of motor vehicles and motorcycles	G46
17	Retail trade	G47
18	Land transport and transport via pipelines, water transport, air transport, warehousing and support activities for transportation and postal and courier activities	H40 to H53
19	Accommodation and food service activities	I55, I56
20	Publishing activities, motion picture and music publishing activities, telecommunications, computer programming and information service activities	J58, J60, J62
21	Finance and insurance	K64
22	Insurance, reinsurance and pension funding, except compulsory social security	K65
23	Activities auxiliary to financial services and insurance activities	K66
24	Real estate activities excluding imputed rents	L68B
25	Imputed rents of owner-occupied dwellings	L68C
26	Legal and accounting activities, activities of head offices, management consultancy activities	M65 and M70
27	Scientific, technical, research and development activities, technical testing and analysis	M71
28	Advertising and market research	M72
29	Other professional, scientific and technical activities, veterinary activities	M73
30	Rental and leasing activities	M74 and M75
31	Employment activities	N77
32	Travel agency, tour operator, reservation service and related activities	N78
33	Security and investigation activities, security guarding and surveillance activities	N79, N80
34	Public administration and defence, compulsory social security	N81, N82
35	Education	O84
36	Human health activities	P85
37	Social work activities	Q86
38	Creative, arts and entertainment activities, gambling and betting activities	Q87 to Q88
39	Activities of membership organisations	R92
40	Activities of membership organisations	R93
41	Repair of computers and personal and household goods	S94
42	Other personal service activities	S95
43	Activities of households as employers, undifferentiated goods and services producing activities of households for own use	S96
44	Activities of extra-territorial organisations and bodies	T

Table 3: Description of the 44 Maltese Economic Sectors Present in the 2010 Micro SAM

Source: (Theuma, 2020).

MacroSAM in Euro Millions (000's)	P	H	F	G	C	E	L	K	T	Tot
Production Activities (P)	3,880.95	3,020.91	0	1,247.73	872.117	8,577.18	0	0	0	17,598.89
Households (H)	0	0	189.79	863.99	0	1,738.25	2,846.27	677.9	0	6,316.21
Enterprises (F)	0	991.14	0	220.08	0	3,982.49	0	2,030.78	0	7,224.54
Government (G)	0	577.48	59.05	0	0	1,123.48	0	251.83	792.74	2,804.55
Capital (C)	0	193.88	-136.78	427.19	0	1,073.94	0	0	0	1,558.18
RoW Imports (E)	7,748.25	1,054.81	7,112.47	32.80	577.46	692.68	0	0	0	17,218.46
Compensation of Employees (L)	2,846.27	0	0	0	0	0	0	0	0	2,846.27
Other Value Added (K)	2,960.51	0	0	0	0	0	0	0	0	2,960.51
Net Taxes on Products and Production (T)	162.908	478	0	12.79	108.64	30.4	0	0	0	792.70
Total (Tot)	17,598.91	6,316.21	7,224.54	2,804.55	1,558.18	17,218.46	2,846.27	2,960.51	792.70	59,320.36

Table 4: Maltese Macro SAM for the reference year of 2010

Source: (Theuma, 2020).

